# Trajectory Design for Alpha CubeSat

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**Summary**

A trajectory design is described for the Alpha CubeSat mission. It satisfies the mission constraints of flight time, ΔV, and final lunar capture orbits. This design can be refined with numerical simulations. The total ΔV needed is 180 m/s, well within the capability of the mission. The total flight time is 315 days.

**Description of Trajectory Design**

Propulsion Capability

The spacecraft for Alpha CubeSat , we label SC, is assumed to have two types of propulsion systems. One is HTSD with available ΔV = .228 km/s and a LTLD with available ΔV = 1.334 km/s. This totals 1.562 km/s. (The LTLD is a Busek BIT-1 ion thruster using iodine – Isp = 1,200 s, Th = .4 micro Newtons. This yields **2.5 m/s per day** of continuous thrusting. It can produce **1.334 km/s** The HTSD thruster has an Isp =200s and Th = 1,400 Newtons. It uses N2O and aluminized paraffin. This yields **228 m/s)**

Starting Conditions (Earth Centered)

Time = 0 (a starting epoch)

Radial distance, rp(E) = 45,000 km

Velocity, vp = 4.19 km/s. This is provided by the launch vehicle for a piggyback payload and not by SC.

Apoapsis Condition(Earth Centered)

rp and vp place SC on a highly eccentric nearly parabolic escape trajectory, TE, from the Earth to an apoapsis distance, ra(E) = 4,000,000 km. This ellipse has an eccentricity at the start of e = .98. TE will be perturbed once SC reaches ra due to solar perturbations, but only slightly.

When SC reaches ra(E), it will be at an approximate apoapsis of an approximate highly eccentric ellipse where the radial velocity is approximately 0. At this location, the trajectory is turning around to return to the Earth.

The time of flight from rp(E) to ra(E) is approximately **166 days.** (see Figure 1)

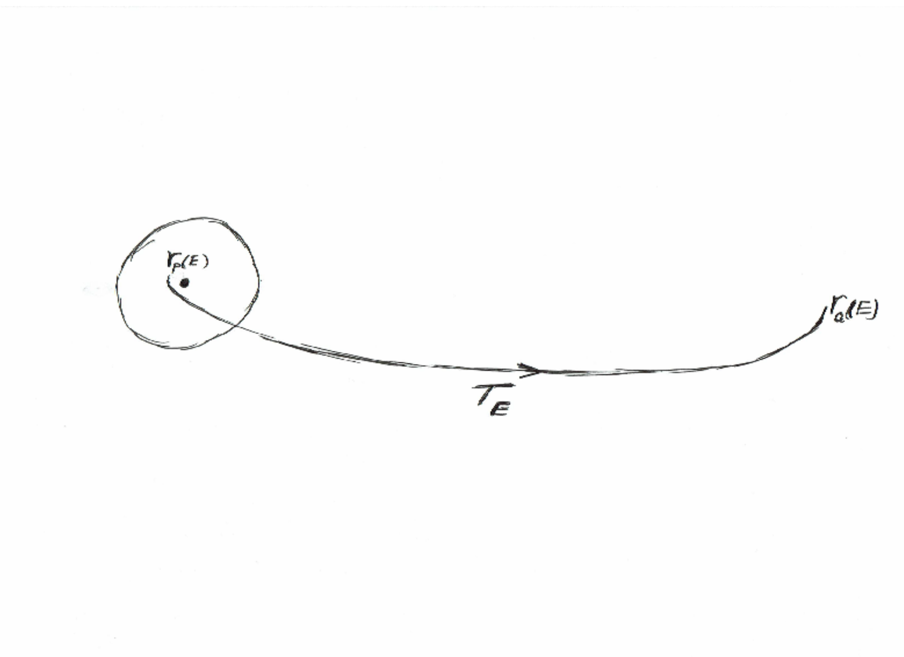


Figure 1. Escape Transfer to 4,000,000 km (inertial coordinates)

<u>Targeting at ra(E) to the Moon</u>

When SC is at ra(E) a maneuver is performed to target the trajectory to reach the Moon on a low energy trajectory that passes near the Earth-Sun L2 point. The energy is adjusted so that upon arrival near L2, SC lies near a stable manifold (a cylindrical tube in position-velocity space) that allows SC to move towards L2 vicinity with minimal energy(velocity), and then exit the L2 neighborhood with minimal velocity near an unstable manifold(another cylindrical tube in position velocity space). These tubes are connected at a halo orbit about L2. This allows the trajectory to move to the Moon with minimal energy and arrive near the Moon with the correct timing. In fact, when SC arrives near the Moon, it does so on a stable manifold to a region about the Moon where ballistic capture occurs – called a Weak Stability Boundary (WSB)[1,3].

The targeting at ra(E) is also done so that upon arrival at lunar periapsis the periapsis altitude, rp(M) is 500 km. The targeting maneuver at ra(E) is estimated to be **ΔV(ra) ≅ 12 m/s**. This achieves both the required plane change and lunar arrival conditions. The fact this maneuver is small is due to the large distance to the Moon and the fact that Earth-Sun L2 region and the lunar arrival state are in the WSB regions of the Earth and Moon, respectively. (see Figure 2)

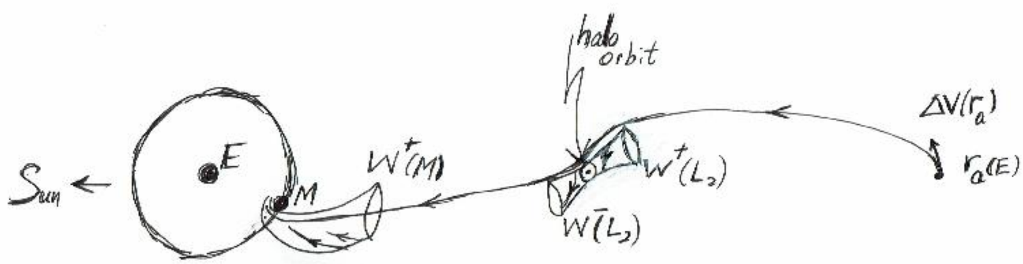


Figure 2. Trajectory from ra(E), passing near the stable manifold ($W^+$) and unstable manifold($W^-$) of $L_2$ . These manifolds exist in position-velocity space and are shown here projected into position space, as an illustration. The trajectory is then guided to the Moon via another stable manifold, $W^+$, of the lunar WSB where lunar capture occurs for 0 ΔV. (Earth-Sun rotating coordinates)

As is described in [2], there exists a special family of orbits about the Moon in the WSB at this altitude, with an apoapsis altitude of ra(M) = 40,000 km. The initial osculating eccentricity is .89. These orbits, which are 500 x 40000 km in altitude are shown to be stable in [2] for at least one month where the orbital elements change by very little.

The remarkable thing about these orbits is that their periapsis exists in the WSB. This means that ballistic capture can occur, so that no ΔV is needed when the trajectory from ra(E) arrives at rp(M). That is, **ΔV1(rp(M)) = 0**. This condition is included when targeting from ra(E).

To satisfy the constraints of the Alpha CubeSat mission, ra(M) is lowered to 10,000 km by performing a maneuver at rp(M) of **ΔV2(rp(M)) = 118 m/s**.   (see Figure 3)

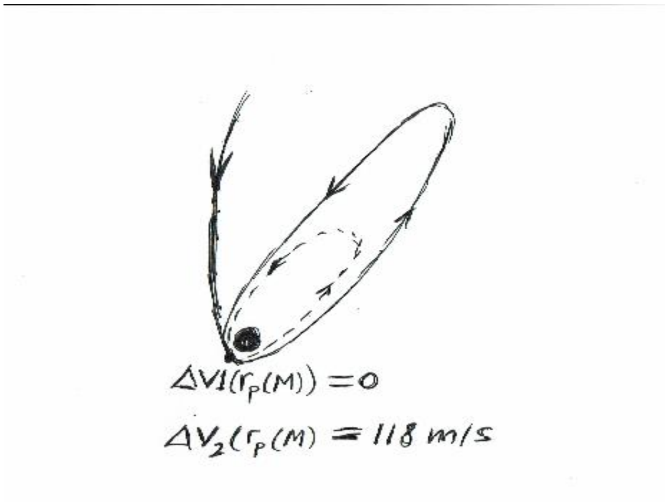


Figure 3.   Arrival of trajectory into ballistic capture at rp(M) so that ΔV1(rp(M)) = 0.   The initial osculating elliptical  orbit has ra(M) = 40,000 km.  A maneuver of ΔV2 = 118 m/s reduces ra to ra(M) = 10,000 km (dashed ellipse).

The time of flight from ra(E) to rp(M) is approximately **149 days.**

Course correction maneuvers may need to be made from the Earth to the Moon. The allocation for these is **ΔV(Corr) = 50 m/s.**

**Summary**

**Total ΔV  =  ΔV(ra(E)) + ΔV1(rp(M)) + ΔV2(rp(M)) + ΔV(Corr) = 180 m/s**

**Total Flight Time  = 315 days**

**References**


1. Belbruno, E. , *Capture Dynamics and Chaotic Motions in Celestial Mechanics*, Princeton University Press, 2005.
2. Belbruno, E., A New Class of Low Energy Lunar Orbits and Mission Applications, *New Trends in Astrodynamics and Applications III*, Volume 886, American Institute of Physics, pp 3-19, 2007.
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4. Post, K.; Belbruno, E.; Topputo, F., Efficient Cis-Lunar Trajectories, in *Proceedings: GLEX-2012.02.3.6x12248*, Washington, D.C., May 22-24, 2012.